LIBERTY PAPER SET

STD. 10 : Mathematics (Standard) [N-012(E)]

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 4**

Section-A

1. (A) 1 **2.** (A) 10 **3.** (C) $\sqrt{(b^2-4ac)^2}$ **4.** (D) 6 π **5.** (D) 2.88 **6.** (C) 70 **7.** infinite **8.** $\frac{1}{2}$ **9.** 2 **10.** $2a$ **11.** $\frac{1}{2}$ **12.** 24 **13.** True **14.** False **15.** True **16.** True **17.** pqr **18.** (0, – 1) **19.** $\frac{\pi R^2 P}{360}$ **20.** $\frac{1}{3}\pi r^2(2r + h)$ **21.** (c) $-\frac{b}{a}$ **22.** (a) $\frac{c}{a}$ **23.** (b) $\sqrt{2}$ **24.** (a) $\sqrt{3}$

Section-B

25. Here, for how many minutes Ravina and Jatin meet at the starting point one has to find the LCM of 18 and 12.

$18 = 2 \times 3^2$

$12 = 2^2 \times 3$

$\therefore$ LCM (18, 12) $= 2^2 \times 3^2 = 36$

$= 36 \times 60$ second

$= 2160$ second

Hence, they will meet again at the stasting point after 2160 second.

26. $x - 3y = 7$ ...(1)

$3x - 3y = 15$ ...(2)

Subtract equation (1) from (2),

$3x - 3y = 15$

$x - 3y = 7$

$- \quad + \quad -$

$2x \quad = 8$

$x = \frac{8}{2}$

$x = 4$

Put $x = 4$ in eq. (1),

$4 - 3y = 7$

$\therefore \; -3y = 7 - 4$

$\therefore \; -3y = 7 - 4$

$\therefore \; -3y = 3$

$\therefore \; 3y = -3$

$\therefore \; y = \frac{-3}{3}$

$y = -1$

$x = 4, y = -1$

27. $3x^2 - 4\sqrt{3}\,x + 4 = 0$

$\therefore\ a = 3,\ b = -4\sqrt{3},\ c = 4$

$\therefore\ b^2 - 4\,ac = \left(-4\sqrt{3}\right)^2 - 4(3)(4) = 48 - 48 = 0$

$\therefore\ b^2 - 4\,ac = 0$

Real roots exist for given equation. and they are equal to each other.

$$\therefore\ x = \frac{-b}{2a} = \frac{-\left(-4\sqrt{3}\right)}{2 \times 3} = \frac{4\sqrt{3}}{6} = \frac{2}{\sqrt{3}}$$

Therefore, roots of given equation = $\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$.

28. Suppose, first number be x,

According the first condition,

second number + first number = 27

$\therefore$ second number + x = 27

$\therefore$ second number = $27 - x$

According to second condition,

First number × second number = 182

$x(27 - x) = 182$

$\therefore\ 27x - x^2 = 182$

$\therefore\ x^2 - 27x + 182 = 0$

$\therefore\ x^2 - 13x - 14x + 182 = 0$

$\therefore\ x(x - 13) - 14\,(x - 13) = 0$

$\therefore\ (x - 13)\,(x - 14) = 0$

$\therefore\ x - 13 = 0$ OR $x - 14 = 0$

$\therefore\ x = 13$ OR $x = 14$

If $x = 13$ i.e. first number = 13

then second number $= 27 - x$

$= 27 - 13$

$= 14$

If first number $= 14$

then, second number $= 27 - x$

$= 27 - 14$

$= 13$

29. Here, $a = 21$, $d = 18 - 21 = -3$

Suppose, n^{th} term $a_n = -81$

$a_n = a + (n - 1)\,d$

$\therefore\ -81 = 21 + (n - 1)\,(-3)$

$\therefore\ -81 = 21 - 3n + 3$

$\therefore\ -81 = 24 - 3n$

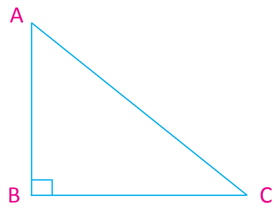
$\therefore\ -27 = 8 - n$

$\therefore\ n = 8 + 27$

$\therefore\ n = 35$

Therefore, the 35^{th} term of the given AP is $-\ 81$

30.



Δ ABC is a right angle triangle, $\angle B = 90°$

$15 \, cot \, A = 8$

$\therefore cot \, A = \frac{8}{15}$

$\therefore \frac{AB}{BC} = \frac{8}{15}$

$\therefore \frac{AB}{8} = \frac{BC}{15} = k$, where k is a positive real number

$\therefore AB = 8k, \; BC = 15k$

According to Pythagoras Theorem,

$AC^2 = AB^2 + BC^2$

$\therefore AC^2 = (8k)^2 + (15k)^2$

$\therefore AC^2 = 64k^2 + 225k^2$

$\therefore AC^2 = 289k^2$

$\therefore AC = 17k$

$\therefore sin \, A = \frac{BC}{AC} = \frac{15k}{17k} = \frac{15}{17}$ and

$cos \, A = \frac{AB}{AC} = \frac{8k}{17k} = \frac{8}{17}$

31. LHS $= \frac{1}{2 - sin^2 \alpha} + \frac{1}{2 + tan^2 \alpha}$

$= \frac{1}{1 + 1 - sin^2 \alpha} + \frac{1}{1 + 1 + tan^2 \alpha}$

$= \frac{1}{1 + cos^2 \alpha} + \frac{1}{1 + sec^2 \alpha}$

$= \frac{1}{1 + cos^2 \alpha} + \frac{1}{1 + \frac{1}{cos^2 \alpha}}$

$= \frac{1}{1 + cos^2 \alpha} + \frac{1}{\frac{cos^2 \alpha + 1}{cos^2 \alpha}}$

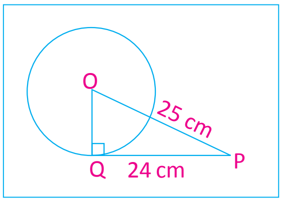
$= \frac{1}{1 + cos^2 \alpha} + \frac{cos^2 \alpha}{1 + cos^2 \alpha}$

$= \frac{1 + cos^2 \alpha}{1 + cos^2 \alpha}$

$= 1$

$=$ RHS

32.



Here, radius = OQ

OP = 25 cm and PQ = 24 cm are given.

OQ ⊥ PQ

∴ For Δ OQP, ∴ $OQ^2 + PQ^2 = OP^2$

∴ $OQ^2 + 24^2 = 25^2$

∴ $OQ^2 + 576 = 625$

∴ $OQ^2 = 625 - 576$

∴ $OQ^2 = 49$

∴ OQ = 7 cm = Radius

∴ Diameter = 2 (Radius)

= 2 (7)

= 14 cm

33. $r = h = 7$ cm

Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 7^2 \times 7$$

$$= 22 \times 49$$

$$= 1078 \text{ cm}^3$$

34. Here highest freq. is 23. So, Mode class = 30 – 35

$l = 30$, $h = 5$, $f_1 = 23$, $f_0 = 21$, $f_2 = 14$

$$Z = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

$$= 30 + \left[\frac{23 - 21}{2(23) - 21 - 14}\right] \times 5$$

$$= 30 + \left[\frac{2}{46 - 35}\right] \times 5$$

$$= 30 + \left[\frac{2 \times 5}{11}\right]$$

$$= 30 + \frac{10}{11}$$

$$Z = 30 + 0.91$$

$$Z = 30.91 \text{ years.}$$

35.

Number of letters (class)	Number of surnames f_i	*cf*
1 – 4	6	6
4 – 7	30	6 + 30 = 36
7 – 10	40	36 + 40 = 76
10 – 13	16	76 + 16 = 92
13 – 16	4	92 + 4 = 96
16 – 19	4	96 + 4 = 100

Here, n = 100

$\therefore \frac{n}{2} = 50$

Since the 50th observation is contained in class 7-10, the median class is 7-10

$\therefore$ l = lower limit of median class = 7

cf = cumulative frequency of class preceding the median class = 36

f = frequency of median class = 40

h = class size = 3

$$\text{Median M} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\therefore \quad M = 7 + \left(\frac{50-36}{40}\right) \times 3$$

$$\therefore M = 7 + \frac{14 \times 3}{40}$$

$$\therefore M = 7 + \frac{21}{20}$$

$$\therefore M = 7 + 1.05$$

$\therefore$ M = 8.05 letters

36. A box contains 90 discs which are number from 1 to 90.

Total number of discs = 90

(i) Suppose A be the event "a perfect square number drawn on disc."

There are 9 perfect square numbers 1, 4, 9, 16, 25, 36, 49, 64 and 81 among 1 to 90.

$\therefore$ The number of outcomes favourable to A = 9

$$\therefore P(A) = \frac{9}{90} = \frac{1}{10}$$

(ii) Suppose B be the event "a perfect cube number drawn on disc."

There are 4 perfect cube numbers 1, 8, 27 and 64 among 1 to 90.

$\therefore$ The number of outcomes favourable to B = 4

$$\therefore P(B) = \frac{4}{90} = \frac{2}{45}$$

37. Here, the number of possible outcomes is six : 1, 2, 3, 4, 5 and 6.

(i) Let E be the event 'getting a number greater than 4' and therefore, the numbers of possible outcomes favourable to E is 2 (5 and 6)

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes}}$$

$$= \frac{2}{6} = \frac{1}{3}$$

(ii) Let F be the event 'getting a number less than or equal to 4'.

The number of outcomes favourable to F is 4 (1, 2, 3, 4).

$$P(F) = \frac{\text{Number of outcomes favourable to F}}{\text{Number of all possible outcomes}}$$

$$= \frac{4}{6} = \frac{2}{3}$$

Section-C

38. Let the quadratic polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$\therefore\ \alpha + \beta = 4 = \frac{4}{1} = \frac{-b}{a}$ and $\alpha\ \beta = 1 = \frac{1}{1} = \frac{c}{a}$

$\therefore\ a = 1,\ \ b = -4,\ c = 1$

So, one quadratic polynomial which fits the given conditions is $x^2 - 4x + 1$. You can check that any other quadratic polynomial which fits these conditions will be of the form $k(x^2 - 4x + 1)$, where k is real.

39. We have,

$a = 1,\ b = 9,\ c = 14$

$\alpha + \beta = -\frac{b}{a}\ \alpha \cdot \beta = \frac{c}{a}$

$\alpha + \beta = -\frac{9}{1}$ $\qquad \alpha \cdot \beta = \frac{14}{1}$

$\alpha + \beta = -9$ and $\alpha \cdot \beta = 14$

$\therefore\ (\alpha + \beta)^2 = (-9)^2$

$\therefore\ \alpha^2 + 2\alpha\beta + \beta^2 = 81$

$\therefore\ \alpha^2 + \beta^2 + 2\alpha\beta = 81$

$\therefore\ \alpha^2 + \beta^2 + 2(14) = 81$

$\therefore\ \alpha^2 + \beta^2 + 28 = 81$

$\therefore\ \alpha^2 + \beta^2 = 81 - 28$

$\therefore\ \alpha^2 + \beta^2 = 53$

40. Here, $a = 8$ years, $d = \frac{4}{12}$ year

$= \frac{1}{3}$ year

AP : 8, $\frac{25}{3}$, $\frac{203}{24}$,...

$S_n = 168$ years.

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore\ 168 = \frac{n}{2}\left[2(8) + (n - 1)\frac{1}{3}\right]$

$\therefore\ 168 \times 2 = n\left[16 + (n - 1)\frac{1}{3}\right]$

$\therefore\ 336 = n\left(\frac{48 + n - 1}{3}\right)$

$\therefore\ 336 \times 3 = 48n + n^2 - \text{n}$

$\therefore\ 1008 = n^2 + 47n$

$\therefore\ n^2 + 47n - 1008 = 0$

$\therefore\ n^2 + 63n - 16n - 1008 = 0$

$\therefore\ n(n + 63) - 16\ (n + 63) = 0$

$\therefore\ (n + 63)\ (n - 16) = 0$

$\therefore\ n + 63 = 0$ OR $n - 16 = 0$

$\therefore\ n = -63$ Not possible So, $n = 16$

Let, $a_n = a + (n - 1)d$

$\therefore\ a_{16} = 8 + (16 - 1)\frac{1}{3}$

$\therefore\ a_{16} = 8 + \text{d}15 \times \frac{1}{3}\text{n}$

$\therefore\ a_{16} = 8 + 5$

$\therefore\ a_{16} = 13$

$\therefore$ So, the elder most student will be of age 13 years.

41. Here, $S_{14} = 1050$, $n = 14$, $a = 10$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore\ S_{14} = \frac{14}{2}[2(10) + (14 - 1)d]$

$\therefore\ \frac{1050 \times 2}{14} = 20 + 13d$

$\therefore\ 13d = 130$

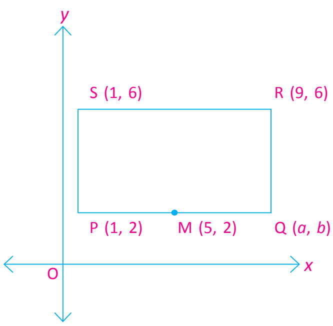
$\therefore\ d = 10$

Now, $a_{20} = a + 19d \quad = 10 + (19 \times 10)$

$= 10 + 190 = 200$

Therefore, 20[th] term is 200.

42.



Here M is midpoint of $\overline{PQ}$.

$\therefore$ Coordinates of M $= \left(\frac{1+a}{2}, \frac{2+b}{2}\right)$

$\therefore\ (5, 2) = \left(\frac{1+a}{2}, \frac{2+b}{2}\right)$

Lets compare both sides.

$5 = \frac{1+a}{2}, \quad 2 = \frac{2+b}{2}$

$\therefore\ 10 = 1 + a,\ 4 = 2 + b$

$\therefore\ a = 10 - 1,\ b = 4 - 2$

$\therefore\ a = 9,\ b = 2$

$\therefore\ Q = (9, 2)$

$SQ = \sqrt{(1-9)^2 + (6-2)^2}$

$\therefore\ SQ = \sqrt{(-8)^2 + (4)^2}$

$\therefore\ SQ = \sqrt{64 + 16}$

$\therefore\ SQ = \sqrt{80}$

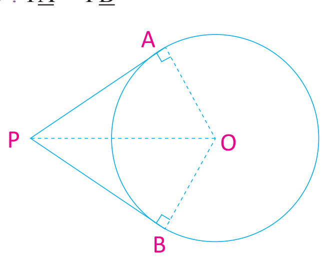
$\therefore\ SQ = \sqrt{16 \times 5}$

$\therefore\ SQ = 4\sqrt{5}$ Unit

43. Given : A circle with centre O, a point P lying outside the circle with two tangents PA and PB on the circle from P.

To prove : PA = PB

Figure :



Liberty

Proof : Join OP, OA and OB. Then ∠OAP and ∠OBP are right angles because these are angles between the radii and tangents and according to theorem 10.1 they are right angles.

Now, in right triangles OAP and OBP,

OA = OB (Radii of the same circle)

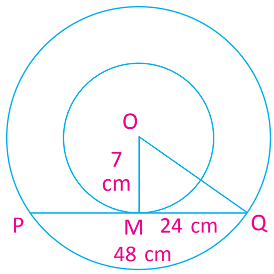
OP = OP (Common)

∠OAP = ∠OBP (Right angle)

Therefore, Δ OAP ≅ Δ OBP (RHS)

This gives, PA = PB (CPCT)

44.



For Δ OMQ, OM ⊥ MQ

∴ M is a midpoint of AQ

∴ $MQ = \frac{1}{2} PQ = \frac{1}{2} (48) = 24$ cm

∴ $OM^2 + MQ^2 = OQ^2$

$7^2 + 24^2 = R^2$

$49 + 576 = R^2$

$625 = R^2$

∴ $R = \sqrt{625}$

∴ R = 25 cm

45. Here, r = 15 cm and θ = 60°

In Δ OAB; ∠O = 60° and OA = OB = 15 cm

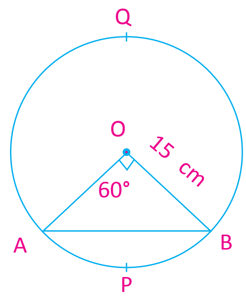
∴ ∠A = ∠B and ∠A + ∠B = 120°

∴ ∠A = ∠B = ∠C = 60°

∴ Δ ABC is an equilateral triangle

In which, length of each side a = 15 cm

Area of equilateral triangle Δ OAB



$= \frac{\sqrt{3}}{4} a^2$

$= \frac{\sqrt{3}}{4} \times 15 \times 15$

$= \frac{1.73}{4} \times 15 \times 15$

$= 97.3125 \text{ cm}^2$

Area of circle= πr^2

$= 3.14 \times 15 \times 15$

$= 706.5 \text{ cm}^2$

Area of minor sector OAPB $= \frac{\pi r^2 \theta}{360}$

$= \frac{3.14 \times 15 \times 15 \times 60}{360}$

$= 117.75 \text{ cm}^2$

Area of minor segment APB

= Area of minor sector OAPB – Area of equilateral triangle Δ OAB

= 117.75 – 97.3125

$= 20.4375 \text{ cm}^2$

46. Total cards in deck = 52

A : card is Red colour king = 2

B : card is not a club = 39

C : card is Queen of heart = 1

We have formula,

$$P(E) = \frac{\text{No. of possible outcomes for given event}}{\text{Total outcomes}}$$

So,

$$P(A) = \frac{2}{52} = \frac{2 \times 1}{26 \times 2} = \frac{1}{26}$$

$$P(B) = \frac{39}{52} = \frac{13 \times 3}{13 \times 4} = \frac{3}{4}$$

$$P(C) = \frac{1}{52}$$

Section-D

47. Let us assume that,

Bhavin's present age = x

Vrutik's present age = y

Five years ago,

Bhavin = $x - 5$

Vrutik = $y - 5$

$\therefore$ As per condition $(x - 5) = 3\ (y - 5)$

$\therefore$ $x - 5 = 3y - 15$

$\therefore$ $x - 3y = -15 + 5 = 0$

$\therefore$ $x - 3y = -10$ ...(1)

10 years from now,

Bhavin will be $x + 10$ & Vrutik will be $y + 10$

As per condition $(x + 10) = 2\ (y + 10)$

$\therefore$ $x + 10 = 2y + 20$

$\therefore$ $x - 2y + 10 - 20 = 0$

$\therefore$ $x - 2y - 10 = 0$ ...(2)

Subtract (2) from (1),

$x - 3y = -10$

$x - 2y = 10$

$- \quad + \quad +$

$\therefore$ $-y + 20 = 0$

$\therefore$ $y = -20$

$y = 20$

Liberty

Put $y = 20$ in eqn (1),

$x - 3y = -10$

$\therefore \quad x - 3(20) = -10$

$\therefore \quad x - 60 = -10$

$\therefore \quad x = -10 + 60$

$\therefore \quad x = 50$

Bhavin's present age = 50 *years*, Vrutik's present age = 20 *years*.

48. Let, the larger no = x,

smaller no = y.

$\therefore \quad x + y = 18$ ...(1)

$x - y = 2$ ...(2)

Add (1) & (2),

$x + y = 18$

$x - y = 2$

$- \quad + \quad -$

$\therefore \; 2x = 20$

$\therefore \; x = 10$

Put $x = 10$ in (1),

$10 + y = 18$

$\therefore \quad y = 18 - 10$

$\therefore \quad y = 18 - 10$

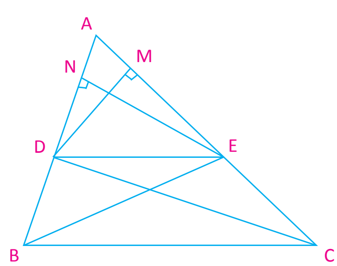
$\therefore \quad y = 8$

Largen no = 10

Smaller no = 8

49. Given: In ABC, a line parallel to side BC intersects AB and AC at D and E respectively.

To prove: $\dfrac{AD}{DB} = \dfrac{AE}{EC}$



Proof : Join BE and CD and also draw DM $\perp$ AC and EN $\perp$ AB.

Then, ADE = $\dfrac{1}{2} \times AD \times EN$,

BDE = $\dfrac{1}{2} \times DB \times EN$,

ADE = $\dfrac{1}{2} \times AE \times DM$ and

DEC = $\dfrac{1}{2} \times EC \times DM$.

$$\therefore \frac{ADE}{BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \qquad ...(1)$$

$$\text{and } \frac{ADE}{DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \qquad ...(2)$$

Now, Δ BDE and Δ DEC are triangles on the same base DE and between the parallel BC and DE.

then, BDE = DEC ...(3)

Hence from eq^n. (1), (2) and (3),

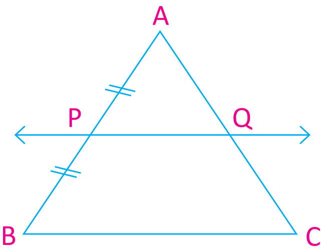
$$\frac{AD}{DB} = \frac{AE}{EC}$$

Liberty

50. Given : In Δ ABC, P is the mid-point of AB side.

Through P a line is drawn parallel to BC to intersect AC at Q.

To prove : Q is the mid-point of AC.



Proof : In Δ ABC, A–P–B and A–Q–C also PQ || BC .

$\therefore \ \dfrac{AP}{PB} = \dfrac{AQ}{QC}$ (∵ theorem : 6.1) ...(1)

Now, P is the mid-point of AB such that AP = PB.

$\therefore \ \dfrac{AP}{PB} = 1$

$\therefore \ \dfrac{AQ}{QC} = 1$ (As per equation (1))

$\therefore$ AQ = QC

$\therefore$ Q is the mid-point of AC.

51. Here AB is the tower C and D are two observation point from tower to respectively 4m and 9m from bottom of tower.

In ΔABC ∠B = 90°

BC = 4*m* and BD = 9*m*

Suppose ∠ACB = θ, then ∠ADB = 90 – θ (∴ Complementry angles)

In Right Tringle ABC

$tan\ \theta = \dfrac{AB}{BC}$

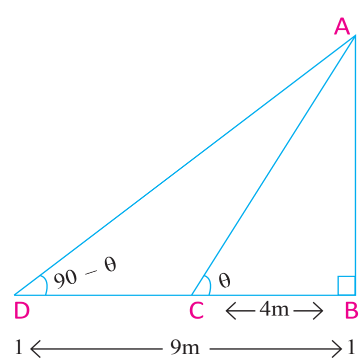
$\therefore\ tan\ \theta = \dfrac{AB}{h}$ ...(1)

In Right Tringle ΔABD

(∴ $tan\ 90 - \theta) = cot\theta$

$\therefore\ tan\ (90 - \theta) = \dfrac{AB}{BD}$

$\therefore\ cot\ \theta = \dfrac{AB}{9}$ ...(2)



From (1) & (2)

$tan\ \theta \cdot cot\ \theta = \dfrac{AB}{4} \times \dfrac{AB}{9}$

$\therefore\ 1 = \dfrac{AB^2}{36}$ (∴ $tan\ \theta \cdot cot\ \theta = 1$)

$\therefore\ 36 = AB^2$

$\therefore$ AB = 6m

$\therefore$ The height of tower = 6 m

Liberty

52. d = 4, $r = \frac{d}{2} = \frac{4}{2} = 2$ m

$h = 7$ m, $l = 3.5$ m

Total area of tent

= Curved surface area of cylinder + Curved surface area of cone

$= 2\pi rh + \pi rl$

$= \pi r\,(2h + l)$

$= \frac{22}{7} \times 2\,[2(7) + 3.5]$

$= \frac{44}{7}\,[14 + 3.5]$

$= \frac{44}{7} \times 17.5$

$= \frac{44 \times 175}{7 \times 10}$

$= \frac{2 \times 22 \times 5 \times 5 \times 7}{7 \times 3 \times 5}$

$= 22 \times 5$

$= 110\ m^2$

cost rate = 100 Rs/m^2

∴ Total cost of 1 tent cenvas = 100 × 110

= 11000 Rs

Total tents = 100

∴ Final total cost of 100 tents = 100 ₹

= 11000 × 100

= 1100000 Rs

Cost contributed by trust = 11,00,000 × 50%

= 5,50,000 Rs

53. h = 14 cm, d = 10 cm

∴ $r = \frac{d}{2} = \frac{10}{2} = 5$ cm

Volume of 1 bottle = $\pi r^2 h$

$= \frac{22}{7} \times 5^2 \times 14$

$= 22 \times 5 \times 5 \times 2$

$= 1100\ cm^3$

= 1100 ml

∴ Volume of 5 bottle = 5 × 1100 ml

= 5500 ml

cost of 100 ml = 10 Rs

cost of 5500 ml $= \frac{5500 \times 10}{100}$

= 550 Rs

Miti will pay 550 Rs to shopkeeper.

54.

Daily pocket allowance (₹)	Number of childern (f_i)	x_i	u_i	f_iu_i
11 – 13	7	12	–4	–28
13 – 15	6	14	–3	–18
15 – 17	9	16	–2	–18
17 – 19	13	18	–1	–13
19 – 21	f	20 = a	0	0
21 – 23	5	22	1	5
23 – 25	4	24	2	8
Total	44 + f	–	–	–64

Mean $\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) h$

$\therefore\ 18 = 20 + \left(\frac{-64}{44 + f}\right) 2$

$\therefore\ 18 - 20 = \frac{-128}{44 + f}$

$\therefore\ -2 = \frac{-128}{44 + f}$

$\therefore\ 44 + f = \frac{-128}{-2}$

$\therefore\ 44 + f = 64$

$\therefore\ f = 64 - 44$

$\therefore\ f = 20$

Hence, the missing frequcncy f is 20.